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Dynamic Response of Non-Uniform Rayleigh Beam Resting on Bi-parametric Elastic Foundation and under the Action of Travelling Distributed Load

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ABSTRACT

This work is carried out to the problem of the dynamic behavior of simply supported Rayleigh beam driven by moving distributed load acting on variable bi-parametric elastic subgrade. The velocity of the motion is assumed constant throughout. General solutions of the thick beams are obtained by the generalized Galerkin's method, Laplace transformation and Convolution Theory for the cases of a load described by the Heaviside function. Expressions for the structural parameters such as axial force, rotatory inertia correction factor, shear force and foundation modulus are obtained and the results displayed in plotted curves.

Key words: Rayleigh beam, non-uniform, bi-parametric, convolution theory, distributed loads

INTRODUCTION

Elastic structures ranging from bridges and highways to space-vehicle are constantly acted upon by moving load (concentrated and distributed) and hence, the problem of analyzing the flexural response of the elastic structures under moving masses. Several investigators in the field of structural dynamics had worked on the reliable method for the accurate determination of the response of the elastic structures when traversed by heavy masses. In most of the studies available in literature, such as the work of Rao[1], Zheng *et al.*[2], Fryba[3] and Oni[4]. The scope has been limited to structural members having uniform sections. The cases where the structure is non-uniform are scanty.

For practical study, in a non-uniform structure, the flexural rigidity and mass per unit length of the beam becomes certain functions of the spatial coordinate x in the model equation. Consequently, upon this the exact solution to the dynamical problem seems impossible as the governing partial differential equation now has variable coefficients. Pertinent among recent researchers when non-uniform structural members have been subjected to heavy masses is the work of Oni[5] who investigated the response of non-uniform beam resting on elastic foundation to several moving masses. The deflection of the non-uniform beam was calculated for several values of foundation moduli and shown graphically as a function of time.

Ogunyebi and Sunday[6] investigated the response of non-uniform beam under tensile stress resting on an elastic foundation. The solution to the fourth order partial differential equation governing the problem was solved when the beam is traversed by mobile distributed loads and from the analysis, response amplitudes of both moving force and moving mass problem decrease with increasing foundation constant. Previous authors that have worked on non-uniform beams include Wu and Dai[7]. They considered the dynamic responses of multi-spam non-uniform beams under moving loads using the transfer matrix method. The dynamic behavior of multi-spam non-uniform beams traversed by a moving load at constant and variable velocities was considered by Dugush and Eisenberger[8].

Abiala and Gbadeyan[9] considered the dynamic behavior of a non-uniform beam resting on a vaiable Winkler foundation and traversed by a uniformly partially distributed moving load. The elastic properties of the beam such as the flexural rigidity, the mass diversity per unit length of the beam, as well as the elastic foundation modulus parameter which are usually assumed constants are hereby expressed as functions of the spatial variables. Recently, Omolofe and Alimi[10] presented the problem of transverse motion of a non-uniform beam with time- dependent boundary conditions when under the action of travelling distributed masses on an elastic foundation using the Mindlin and Goodman's method.

In above studies, structural members are modeled as moving distributed and concentrated loads on an elastic foundation. A more practical situation where two-parameter elastic foundation appears in the model equation has been neglected. This may due to amount of mathematical complexity involved in solving the governing equation. Thus, this work is concerned with the flexural behavior of non-uniform simply supported Rayleigh beam subjected to moving distributed loads travailing on variable bi-parametric elastic foundation at constant speed. Numerical analysis will be presented for both moving distributed force and moving distributed mass solutions.

MATERIALS AND METHODS

Mathematical formulation: The transverse vibration of simply supported pre-stressed non-uniform Rayleigh beam resting on variable Pasternak foundation and traversed by moving distributed masses was governed by the fourth order partial differential equation is given by:

$$\frac{\partial^2}{\partial x^2}\left[EJ(x)\frac{\partial^2 V_n(x,t)}{\partial x^2}\right] - N\frac{\partial^2 V_n(x,t)}{\partial x^2} + \mu^*(x)\frac{\partial^2 V_n(x,t)}{\partial t^2} - \mu_0(x)R^0\frac{\partial^4 V_n(x,t)}{\partial x^2\partial t^2}$$
$$+MH(x-ct)\left[\frac{\partial^2}{\partial t^2} + 2c\frac{\partial^2}{\partial x\partial t} + c^2\frac{\partial^2}{\partial x^2}\right]V_n(x,t) + F_k(x)V_n(x,t) = MH(x-ct) \quad (1)$$

In this system, we shall take variable Pasternak elastic foundation $F_k(x)V_n(x,t)$ to be of the form:

$$F_k(x)V_n(x,t) = S(x)V_n(x,t) - K'(x)\frac{\partial V_n(x,t)}{\partial x} - K(x)\frac{\partial^2 V_n(x,t)}{\partial x^2} \quad (2)$$

where S (x) is the variable foundation stiffness and K (x) is the variable shear modulus.

An example of variable Pasternak elastic foundation in Dugush and Eisenberger[8] and was adopted. J (x) and μ* (x) are taken to be of the form:

$$J(x) = J_o\left(1 + \frac{\sin \pi x}{L}\right)^3 \quad (3)$$

$$\mu^*(x) = \mu_o\left(1 + \frac{\sin \pi x}{L}\right) \quad (4)$$

where J_o and μ_0 are constants.

At this juncture, the boundary conditions for the dynamical system were arbitrary and the initial conditions without any loss of generality are taken to be:

$$V_n(x,0) = 0 = \frac{\partial V_n(x,0)}{\partial t} \quad (5)$$

Substituting Eq. 2, 3 and 4 into Eq. 1, one obtain

$$\sum_{m=1}^{n} EJ_0\frac{\partial^2}{\partial x^2}\left[\left(1+\sin\frac{\pi x}{L}\right)^3\frac{\partial^2}{\partial x^2}V_n(x,t)\right] - N\frac{\partial^2}{\partial x^2}V_n(x,t) + \mu_0\left(1+\sin\frac{\pi x}{L}\right)\frac{\partial^2}{\partial t^2}V_n(x,t)$$
$$-R^0\mu_0\left(1+\sin\frac{\pi x}{L}\right)\frac{\partial^4}{\partial x^2\partial t^2}V_n(x,t) + S_0\left(4x-3x^2+x^3\right)V_n(x,t) + K_0\left(-13+12x-3x^2\right)\frac{\partial}{\partial x}V_n(x,t)$$
$$+K_0\left(12+3x-6x^2 1x^3\right)\frac{\partial^2}{\partial x^2}V_n(x,t) + MH(x-ct)\left[\frac{\partial^2}{\partial t^2} + 2c\frac{\partial^2}{\partial x\partial t} + c^2\frac{\partial^2}{\partial x^2}\right]V_n(x,t) = MgH(x-ct) \quad (6)$$

Equation 6 is the fourth order partial differential equation governing the motion of a non-uniform pre-stressed simply supported Rayleigh beam resting on a variable bi-parametric elastic foundation at uniform velocity.

Methodology: The fourth order partial differential Eq. 6 has both singular and variable coefficients. Evidently, a closed form solution does not exist. Therefore, an elegant method of solution was given to solve the differential equation of motion. The approach involves expressing the Heaviside function as a Fourier Sine series and then reducing the modified form of the fourth order partial differential equation above using the method of Generalized Galerkin extensively discussed in Oni and Ogunyebi[11]. To this end, the coupled fourth order partial differential Eq. 6 was then solved by Struble's technique.

The versatile technique requires that the solution of Eq. 6 takes the form:

$$V_n(x,t)=\sum_{m=1}^{\infty} Z_m(t)U_m(x) \qquad (7)$$

where, U_m (x) was chosen such that the desired boundary conditions were fulfilled.

Equation 7 when substituted into Eq. 6 and after further simplification yields

$$\begin{aligned}
&\sum_{m=1}^{n}\Bigg\{\frac{EJ_0}{4}\Bigg[10U_m^{IV}(x)+15\sin\frac{\pi x}{L}U_m^{IV}(x)+30\frac{\pi}{L}\cos\frac{\pi x}{L}U_m^{III}(x)-\\
&15\frac{\pi^2}{L^2}\sin\frac{\pi x}{L}U_m^{II}(x)-6\cos 2\frac{\pi x}{L}U_m^{IV}(x)\\
&+24\sin 2\frac{\pi x}{L}U_m^{III}(x)+24\frac{\pi^2}{L^2}\cos\frac{\pi x}{L}U_m^{II}(x)-\\
&\sin 3\frac{\pi x}{L}U_m^{IV}(x)-6\frac{\pi}{L}\cos\frac{\pi x}{L}U_m^{III}(x)+9\frac{\pi^2}{L^2}\sin 3\frac{\pi x}{L}U_m^{II}(x)\Bigg]\ddot{Z}_m(t)\\
&-NU_m''(x)Y_m(t)+\mu_0\left[U_m(x)+\sin\frac{\pi x}{L}U_m(x)\right]\ddot{V}_m(t)-\\
&R^0\mu_0\left[U_m^{II}(x)+\sin\frac{\pi x}{L}U_m^{II}(x)\right]\dot{Z}_m(t)\\
&-S_0\left[4xU_m(x)-3x^2U_m(x)+x^3U_m(x)\right]V_m(t)+\\
&K_0\left[-13U_m'(x)+12xU_m'(x)-13x^2U_m'(x)\right]Z_m(t)+MH(x-ct)\\
&\left[U_m(x)\ddot{V}_m(t)+2cU_m'(x)\dot{V}_m(t)+c^2U_m''(x)V_m(t)\right]-MgH(x-ct)\Bigg\}=0
\end{aligned} \qquad (8)$$

In order to determine Z_m (t), it was required that the expression on the left hand side of Eq. 8 be orthogonal to function U_k (t). Thus,

$$\begin{aligned}
&\sum_{m=1}^{n}\Bigg\{\frac{EJ_0}{4}\Bigg[10U_m^{IV}(x)+15\sin\frac{\pi x}{L}U_m^{IV}(x)+30\frac{\pi}{L}\cos\frac{\pi x}{L}U_m^{III}(x)\\
&-15\frac{\pi^2}{L^2}\sin\frac{\pi x}{L}U_m^{II}(x)-6\cos 2\frac{\pi x}{L}U_m^{IV}(x)\\
&+24\sin 2\frac{\pi x}{L}U_m^{III}(x)+24\frac{\pi^2}{L^2}\cos\frac{\pi x}{L}U_m^{II}(x)-\sin 3\frac{\pi x}{L}U_m^{IV}(x)-\\
&6\frac{\pi}{L}\cos\frac{\pi x}{L}U_m^{III}(x)+9\frac{\pi^2}{L^2}\sin 3\frac{\pi x}{L}U_m^{II}(x)\Bigg]\ddot{Z}_m(t)\\
&-NU_m''(x)Y_m(t)+\mu_0\left[U_m(x)+\sin\frac{\pi x}{L}U_m(x)\right]\ddot{Y}_m(t)-\\
&R^0\mu_0\left[U_m^{II}(x)+\sin\frac{\pi x}{L}U_m^{II}(x)\right]\dot{Z}_m(t)\\
&-S_0\left[4xU_m(x)-3x^2U_m(x)+x^3U_m(x)\right]Y_m(t)+\\
&K_0\left[-13U_m'(x)+12xU_m'(x)-13x^2U_m'(x)\right]Z_m(t)\\
&+MH(x-ct)\begin{bmatrix}U_m(x)\ddot{Y}_m(t)+2cU_m'(x)\dot{Y}_m(t)+\\ c^2U_m''(x)Y_m(t)\end{bmatrix}-MgH(x-ct)\Bigg\}U_k(x)dx=0
\end{aligned} \qquad (9)$$

An appropriate selection of functions for beam problems are beam mode shapes. Thus, the mth normal mode of vibration of a non-uniform Rayleigh beam

$$U_m(x)=\sin\frac{\lambda_m x}{L}+A_m\cos\frac{\lambda_m x}{L}+B_m\sinh\frac{\lambda_m x}{L}+C_m\cosh\frac{\lambda_m x}{L} \qquad (10)$$

was chosen such that the boundary conditions were satisfied. In Eq. 10, λ_m is the mode frequency, A_m,B_m,C_m are constants which are obtained by substituting (10) into the appropriate boundary conditions.

Neglecting the summation sign in Eq. 9 and assumed that the beam has a simple supports at both ends i.e., x = 0 and x = L. Thus, from Eq. 10, Eq. 7 becomes

$$\begin{aligned}
&\sum_{m=1}^{n}\Big\{\Big[A_1(m,k)+A_2(m,k)-R^0\big(A_3(m,k)+A_4(m,k)+A_5(m,k)\big)\Big]\ddot{Z}_m(t)\\
&\Big\{Q_A\Big[10A_6(m,k)+15A_7(m,k)-6A_8(m,k)-A_9(m,k)+\frac{6\pi}{L}\big(A_{10}(m,k)\\
&+4A_{11}(m,k)-A_{12}(m,k)\big)+\frac{6\pi^2}{L^2}\big(3A_{13}(m,k)+8A_{14}(m,k)-5A_{15}(m,k)\big)\Big]-Q_BA_{16}(m,k)\Big\}\dot{Z}_m(t)\\
&+MH(x-ct)\Big[U_m(x)\ddot{Y}_m(t)+2cU_m'\dot{Y}_m(t)+c^2U_m''\dot{Y}_m(t)\Big]-MgH(x-ct)\Big\}U_k(x)dx=0
\end{aligned} \qquad (11)$$

Substituting (11) into (9), we have

$$\begin{aligned}
&\sum_{m=1}^{n}\Big\{\Big[A_1(m,k)+A_2(m,k)-R^0\big(A_3(m,k)+A_4(m,k)+A_5(m,k)\big)\Big]\ddot{Z}_m(t)\\
&\Big\{Q_A\Big[10A_6(m,k)+15A_7(m,k)-6A_8(m,k)-A_9(m,k)+\frac{6\pi}{L}\big(A_{10}(m,k)\\
&+4A_{11}(m,k)-A_{12}(m,k)\big)+\frac{6\pi^2}{L^2}\big(3A_{13}(m,k)+8A_{14}(m,k)-5A_{15}(m,k)\big)\Big]-Q_BA_{16}(m,k)\Big\}\dot{Z}_m(t)\\
&+MH(x-ct)\Big[U_m(x)\ddot{Y}_m(t)+2cU_m'\dot{Y}_m(t)+c^2U_m''\dot{Y}_m(t)\Big]-MgH(x-ct)\Big\}U_k(x)dx=0
\end{aligned} \qquad (12)$$

Where:

$$Q_A=\frac{EJ_0}{4\mu_0};\ \frac{N}{\mu_0}=Q_B$$

when use is made of the Fourier Sine series representation for the Heaviside function given as

$$H(x-ct)=\frac{1}{4}+\frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\sin((2n+1)\pi(x-ct))}{2n+1},\quad 0<x<1 \qquad (13)$$

Using Eq. 13 in 12 and after some arrangements, one obtains

$$\begin{aligned}
&\sum_{\substack{m=1\\m\neq k}}^{n}\Bigg\{\Bigg(\frac{L}{2}+\frac{4mnkL}{\pi\left[(n-k)^2-m^2\right]\left[(n+k)^2-m^2\right]}+\frac{R^0m^2\pi^2}{2L}+\frac{4m^3\pi nkR^0}{\left[(n-k)^2-m^2\right]\left[(n+k)^2-m^2\right]}\\
&+\frac{R^0 2m^2\pi^2\left(n+k^2-m^2\right)}{\left[(n+k)^2-m^2\right]\left[(n-k)^2-m^2\right]}\Bigg)\ddot{Z}_m(t)+\Bigg(\frac{EJ_0}{4\mu_0}\Bigg[\frac{5m^4\pi^4}{L^3}+\frac{8\pi^3nk\left(7m^5+12m^4-12m^3\right)}{L^3\left[(n+k)^2-m^2\right]\left[(n-k)^2-m^2\right]}\\
&+\frac{12\pi^5\left(n^2+k^2-m^2\right)\left(m^5-4m^4-4m^3\right)}{L^3\left[(n+k)^2-m^2\right]\left[(n-k)^2-m^2\right]}\Bigg]-\frac{Nm^2\pi^2}{2\mu_0L}-\frac{8S_0L^2mk}{\mu_0\pi^2(m+k)^2(m-k)^2}-\frac{4L^3S_0}{\mu_0(m-k)^3\pi^3}\\
&-\frac{2K_0m(m+6L)}{\mu_0(k^2-m^2)}-\frac{6K_0L^3m\pi}{\mu_0}\left[\frac{8k^3m+8km^3}{(k-m)^4(k+m)^4}\right]-\frac{m^2\pi^2}{L}+\frac{12m^3k(1-4L)}{(m+k)^2(m-k)^2}+\frac{4m^2\pi L}{(m-k)^3}\Bigg)\Bigg)Z_m(t)\\
&+\varepsilon_0\Bigg[\Bigg(\frac{L}{8}+\frac{1}{\pi}\sum_{n=0}^{\infty}\cos\frac{(2n+1)\pi ct}{2n+1}\left[H_1-H_2\right]-\frac{1}{\pi}\sum_{n=0}^{\infty}\sin\frac{(2n+1)\pi ct}{2n+1}\left[H_3-H_4\right]\Bigg)\ddot{Z}_m(t)\\
&+2C\Bigg(\frac{-m^2}{2(m^2-k^2)}+\frac{m}{2\pi}\sum_{n=0}^{\infty}\cos\frac{(2n+1)\pi ct}{2n+1}\left[H_5-H_6\right]-\frac{L}{2\pi^2}\sum_{n=0}^{\infty}\sin\frac{(2n+1)\pi ct}{2n+1}\left[H_7-H_8\right]\Bigg)\dot{Z}_m(t)\\
&+C^2\Bigg(\frac{-m^2\pi^2}{8L}-\frac{-m^2\pi}{L^2}\sum_{n=0}^{\infty}\sin\frac{(2n+1)\pi ct}{2n+1}\left[H_9-H_{10}\right]-\frac{m^2\pi^2}{L^2}\sum_{n=0}^{\infty}\sin\frac{(2n+1)\pi ct}{2n+1}\left[H_{11}-H_{12}\right]\Bigg)Z_m(t)\Bigg]\\
&=\frac{PL}{\mu m\pi}\left[-\cos m\pi+\cos\frac{m\pi ct}{L}\right]
\end{aligned}\tag{14}$$

Where:

$$\begin{aligned}
H_1&=H_1(n,m,k)=\frac{(2n+1)L^2}{2\pi}\cdot\frac{(-1)^{m+k}\cos(2n+1)\pi L-1}{\left[(2n+1)L\right]^2-\left[m+k\right]^2}\\
H_2&=H_2(n,m,k)=\frac{(2n+1)L^2}{2\pi}\cdot\frac{(-1)^{m-k}\cos(2n+1)\pi L-1}{\left[(2n+1)L\right]^2-\left[m-k\right]^2}\\
H_3&=H_3(n,m,k)=\frac{(2n+1)L^2}{2\pi}\cdot\frac{(-1)^{m-k}\sin(2n+1)}{\left[(2n+1)L\right]^2-\left[m-k\right]^2}\\
H_4&=H_4(n,m,k)=\frac{(2n+1)L^2}{2\pi}\cdot\frac{(-1)^{m+k}\sin(2n+1)}{\left[(2n+1)L\right]^2-\left[m+k\right]^2}\\
H_5&=H_5(n,m,k)=\frac{(m+k)(-1)^{m+k}\sin(2n+1)\pi L}{\left[(2n+1)L\right]^2-\left[m+k\right]^2}\\
H_6&=H_6(n,m,k)=\frac{(m-k)(-1)^{m-k}\sin(2n+1)\pi L}{\left[(2n+1)L\right]^2-\left[m-k\right]^2}\\
H_7&=H_7(n,m,k)=(m+k)\frac{\left[(-1)^{m+k}\cos(2n+1)\pi L-1\right]}{\left[(2n+1)L\right]^2-\left[m+k\right]^2}\\
H_8&=H_8(n,m,k)=(m-k)\frac{\left[(-1)^{m-k}\cos(2n+1)\pi L-1\right]}{\left[(2n+1)L\right]^2-\left[m-k\right]^2}\\
H_9&=H_9(n,m,k)=\frac{(2n+1)L^2}{2\pi}\cdot\frac{(-1)^{m+k}\cos(2n+1)\pi L-1}{\left[(2n+1)L\right]^2-\left[m+k\right]^2}\\
H_{10}&=H_{10}(n,m,k)=\frac{(2n+1)L^2}{2\pi}\cdot\frac{(-1)^{m-k}\cos(2n+1)\pi L-1}{\left[(2n+1)L\right]^2-\left[m-k\right]^2}\\
H_{11}&=H_{11}(n,m,k)=\frac{(2n+1)L^2}{2\pi}\cdot\frac{(-1)^{m-k}\sin(2n+1)\pi L}{\left[(2n+1)L\right]^2-\left[m-k\right]^2}\\
H_{12}&=H_{12}(n,m,k)=\frac{(2n+1)L^2}{2\pi}\cdot\frac{(-1)^{m+k}\sin(2n+1)\pi L}{\left[(2n+1)L\right]^2-\left[m+k\right]^2}
\end{aligned}\tag{15}$$

Equation 14 represents the transformed equation governing non-uniform finite simply supported Rayleigh beam resting on variable bi-parametric subgrade under moving distributed masses. Consequently, two cases of Eq. 14 were considered namely: The moving distributed force and the moving distributed mass problem.

Pre-stressed simply supported rayleigh beam traversed by distributed forces: In this section, only the force effect of the moving distributed loads was considered by setting $\lambda_0 = 0$. To this end, Eq. 14 reduces to

$$\ddot{Z}_m(t)+\alpha_{mf}^2V_m(t)=\frac{PL}{\mu_0m\pi F^*}\left[-(-1)^m+\cos\frac{m\pi ct}{L}\right]\tag{16}$$

Where:

$$\begin{aligned}
\alpha_{mf}^2=&\frac{5EJ_om^4\pi^4}{4\mu_oL^3F^*}+\frac{4EJ_o\pi^3nk(7m^5+12m^4-12m^3)}{\mu_oL^3F^*\left[(n+k)^2-m^2\right]\left[(n-k)^2-m^2\right]}-\\
&\frac{4EJ_o\pi^5(n^2+k^2-m^2)(m^5-4m^4-4m^3)}{\mu_oL^3F^*\left[(n+k)^2-m^2\right]\left[(n-k)^2-m^2\right]}\\
&-\frac{Nm^2\pi^2}{2\mu_0LF^*}-\frac{8S_0L^2mk}{\mu_0\pi^2F^*(m+k)^2(m-k)^2}-\frac{4S_0L^3}{\mu_0\pi^3F^*(m-k)^3}-\\
&\frac{m^2\pi^2}{LF^*}-\frac{2k_0m(m+6L)}{\mu_0F^*(m^2-k^2)}-\frac{6K_0L^3m\pi}{\mu_0F^*}\\
&\left[\frac{8K_0m+8km^3}{(m-k)^4(m+k)^4}\right]+\frac{4m^2\pi L}{F^*(m-k)^3}+\frac{12m^3k(1-4L)}{F^*(m+k)^2(m-k)^2}
\end{aligned}\tag{17}$$

and

$$\begin{aligned}
F^*=&\frac{L}{2}+\frac{4mnkL}{\pi\left[(n-k)^2-m^2\right]\left[(n+k)^2-m^2\right]}+\frac{R^0m^2\pi^2}{2L}+\\
&\frac{4m^3\pi nk}{\left[(n-k)^2-m^2\right]\left[(n+k)^2-m^2\right]}+\frac{2m^2\pi^2(n^2+k^2-m^2)}{\left[(n-k)^2-m^2\right]\left[(n+k)^2-m^2\right]}
\end{aligned}\tag{18}$$

To this end, a modification of Struble's asymptotic technique described in Oni and Ogunyebi[11] was employed in conjunction with Laplace and convolution theory to give an expression for $Z_m(t)$.
which on inversion yields,

$$V_n(x,t)=\sum_{m=1}^{n}\frac{PL}{\mu_0m\pi F^*}\left[\frac{\cos q_mt-\cos\alpha_{mf}t}{\alpha_{mf}^2-q_m^2}+\frac{(1-\cos\alpha_{mf}t)}{\alpha_{mf}}\right]\times\frac{\sin mx}{L}\tag{19}$$

Equation 19 is the transverse response of moving distributed force of a simply supported non-uniform Rayleigh beam resting on variable bi-parametric elastic foundation at constant speed.

Prestressed simply supported rayleigh beam traversed by distributed masses: When $\lambda_0 \neq 0$ in Eq. 14, the solution to the entire equation was required where both force effect and mass effects are considered. This is called moving distributed mass problem and by the asymptotic method due to Struble, it is straight forward to show that

$$\frac{d^2}{dt^2}Z_m(t)+\alpha_{mm}^2 Z_m(t)=\frac{PL}{\mu_0 m\pi F^*}\left[-(-1)^m+\cos\frac{m\pi ct}{L}\right] \tag{20}$$

Where:

$$\alpha_{mm}=\alpha_{mf}\left[1-\frac{E_2}{16F^*}\left(\frac{c^2m^2\pi^2}{\alpha_{mf}^2}-L\right)\right] \tag{21}$$

is called the modified frequency corresponding to the frequency of the free system due to the presence of the moving mass.

Solving Eq. 20 in conjunction with the initial conditions gives an expression for $Z_m(t)$ which when inverted gives

$$V_n(x,t)=\sum_{m=1}^{n}\frac{E_2L^2g}{\mu_0 m\pi F^*}\left[\frac{\cos q_m t-\cos\alpha_{mm}t}{\alpha_{mm}^2-q_m^2}+\frac{(1-\cos\alpha_j t)}{\alpha_{mm}}\right]\times\frac{\sin mx}{L} \tag{22}$$

Equation 23 represents the transverse displacement response of moving distributed mass at constant velocity of a simply supported non-uniform Rayleigh beam resting on variable bi-parametric elastic foundation.

Comments on closed form solution: This section seeks to establish the conditions under which the dynamical system grows without bound. Therefore, it was pertinent to discuss the issue of resonance as design engineers in the area of constructions engineering find this useful.

Equation 19 clearly shows that the finite non-uniform simply supported Rayleigh beam resting on a variable bi-parametric elastic subgrade and traversed by a moving distributed force reaches a state of resonance whenever

$$\alpha_{mf}=\frac{m\pi c}{L} \tag{23}$$

while Eq. 22 shows that the same beam under the action of moving mass will experience resonance effect whenever

$$\alpha_{mm}=\frac{m\pi c}{L} \tag{24}$$

from Eq. 21

$$\alpha_{mm}=\alpha_{mf}\left[1-\frac{E_2}{16F^*}\left(\frac{c^2m^2\pi^2}{\alpha_{mf}^2}-L\right)\right] \tag{25}$$

which implies

$$\alpha_{mm}=\alpha_{mf}\left[1-\frac{E_2}{16F^*}\left(\frac{c^2m^2\pi^2}{\alpha_{mf}^2}-L\right)\right]=\frac{m\pi c}{L} \tag{26}$$

It can be seen that, for the same natural frequency, the critical speed for the system consisting of a finite non-uniform simply supported Rayleigh beam resting on a variable bi-parametric elastic foundation and traversed by moving distributed force with uniform speed is greater than that of moving distributed mass problem. Thus for the same natural frequency, resonance is reached earlier in the moving distributed mass system than in the moving distributed force system[11].

RESULTS

To illustrate the foregoing analysis, the finite non-uniform simply supported Rayleigh beam resting on a variable bi-parametric elastic foundation of length L = 12.192 m was considered. Furthermore, homogenous beam of modulus of elasticity $E = 3.1\times10^{10}$ N m^{-2}, the moment of inertia $J = 2.87698\times10^{-3}$ m^4, velocity 8.123 m sec^{-1} and the mass per unit length of the beam $\mu_0 = 2758.291$ kg m^{-1} are respectively chosen. The results of the dynamic behaviour of the finite simply supported non-uniform Rayleigh beam are presented on the various graphs below.

Figure 1 and 2 displays the effects of axial force N on the flexural vibrations of a non-uniform simply supported uniform

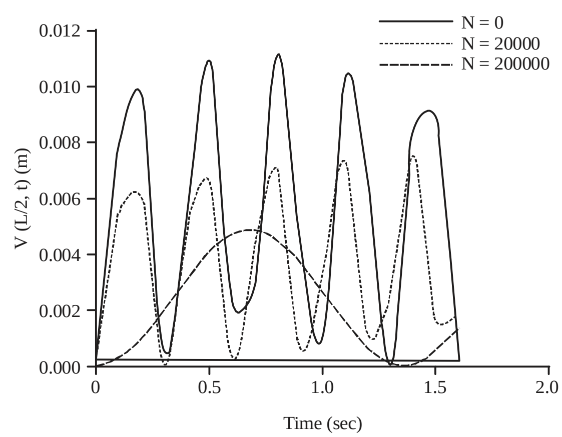

Fig. 1: Deflection profile of a simply supported non-uniform rayleigh beam on variable Pasternak foundation and traversed by moving distributed force for So = 3000, Ko = 10000, Ro = 0.2 and various values of No

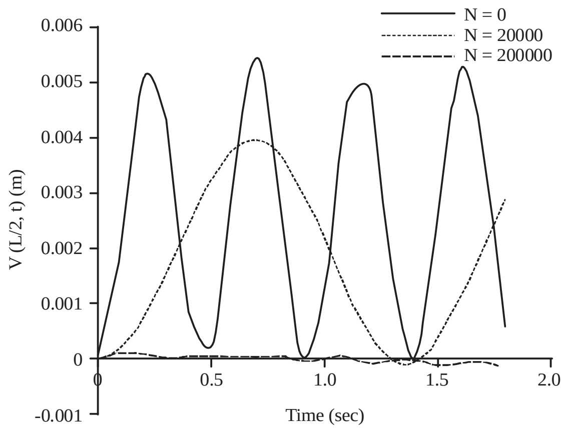

Fig. 2: Displacement response of a simply supported non-uniform Rayleigh beam on variable Pasternak foundation and traversed by moving distributed mass for So = 3000, Eo = 0.5, Ko = 10000, Ro = 0.2 and various values of No

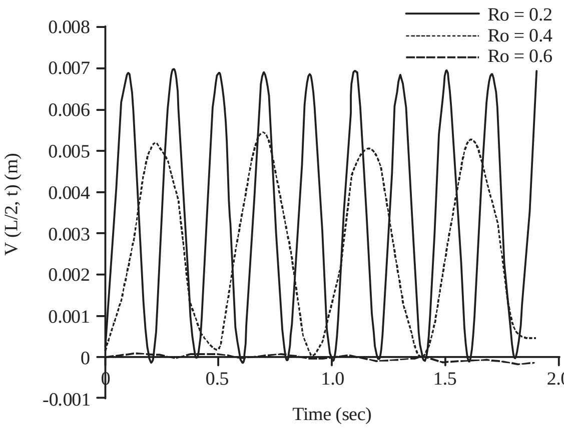

Fig. 4: Deflection profile of a simply supported non-uniform Rayleigh beam on variable Pasternak foundation and traversed by moving distributed mass for N = 2000, Eo = 0.5, Ko = 10000, So = 3000 and various values of Ro

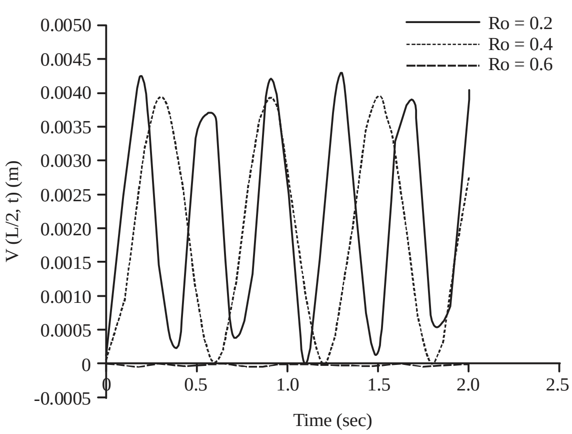

Fig. 3: Transverse displacement of a simply supported non-uniform Rayleigh beam on variable Pasternak foundation and traversed by moving distributed force for N = 2000, Ko = 10000, So = 3000 and various values of Ro

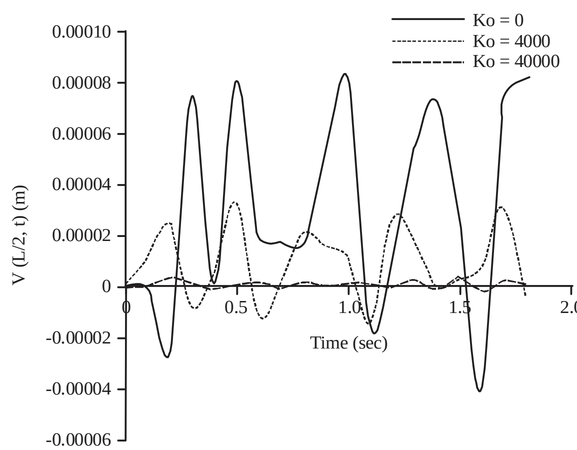

Fig. 5: Displacement response of a simply supported non-uniform Rayleigh beam on variable Pasternak foundation and traversed by moving distributed force for N = 2000, So = 3000, Ro = 0.2 and various values of Ko

Rayleigh beam under the action of moving distributed forces moving at constant velocity in both cases of moving distributed force and moving distributed mass respectively. The graphs show that the response amplitude decreases as the value of the axial force increases.

Figure 3 and 4 shows the effect of rotatory inertia Ro on the transverse displacement of the non-uniform simply supported Rayleigh beam in both cases of moving distributed force and moving distributed mass respectively. The curves show that the response amplitude decreases as the value of the rotatory inertia correction factor increases.

Figure 5 and 6 displays the effects of foundation stiffness So on deflection amplitude of the non-uniform simply supported Rayleigh beam transverse by moving distributed force and moving distributed mass for So = 30000, N = 2000 and Ro = 0.2 respectively. It can be seen from the graphs that the response amplitude decreases as the value of the So increases.

Figure 7 and 8 displays the deflection of the shear modulus Ko for the non-uniform Rayleigh beam transverse by moving distributed force and moving distributed mass. As the value of

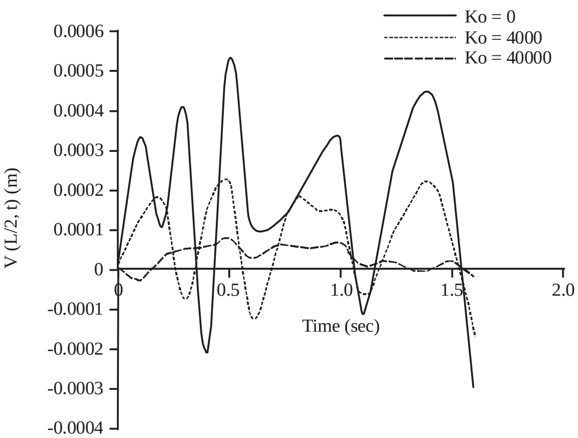

Fig. 6: Transverse displacement of a simply supported non-uniform Rayleigh beam on variable Pasternak foundation and traversed by moving distributed mass for N = 2000, Eo = 0.5, So = 3000, Ro = 0.2 and various values of Ko

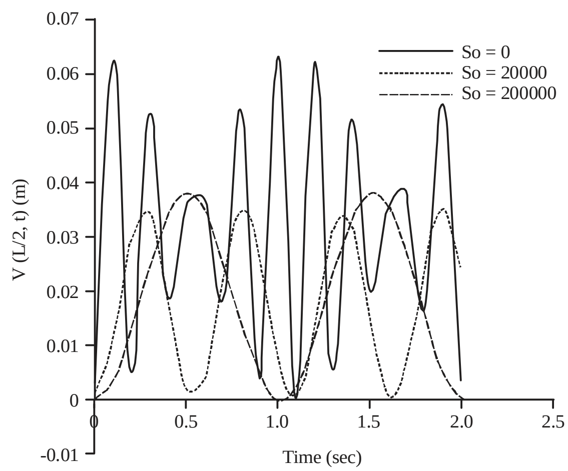

Fig. 7: Deflection profile of a simply supported non-uniform Rayleigh beam on variable Pasternak foundation and traversed by moving distributed force for N = 10000, Ko = 10000, Ro = 0.2 and various values of So

Ko increases, the response amplitude of the beam for moving distributed force and moving distributed mass decreases.

Figure 9 displays the deflection profile of the mass ratio for the non-uniform Rayleigh beam traversed by moving distributed mass. As the value of E_0 increases, response amplitude of the beam for the moving distributed mass decreases.

Figure 10 compares the displacement curves of the moving distributed force and moving distributed mass for a simply supported non-uniform Rayleigh beam with Ko = 2000 N m^{-2}. N = 2000 N m^{-2}, Ro = 0.2 and So = 30000 n m^{-2}. Clearly, the

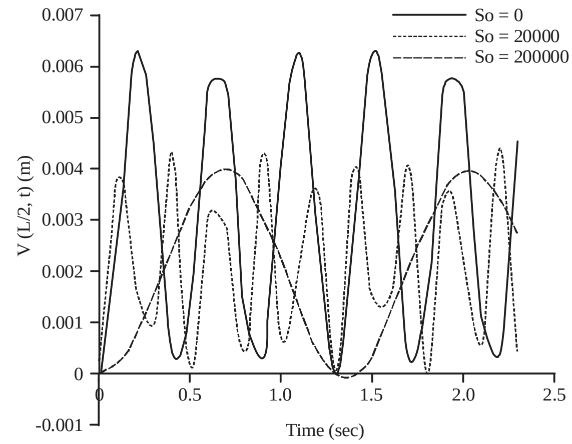

Fig. 8: Displacement response of a simply supported non-uniform Rayleigh beam on variable Pasternak foundation and traversed by moving distributed mass for N = 10000, Eo = 0.5, Ko = 10000, Ro = 0.2 and various values of So

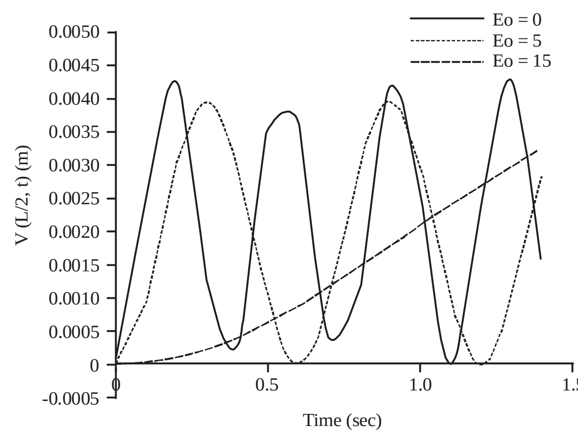

Fig. 9: Transverse displacement of a simply supported non-uniform Rayleigh beam on variable Pasternak foundation and traversed by moving distributed mass for N = 10000, So = 30000, Ko = 2000, Ro = 0.2 and various values of Eo

response amplitude of a moving distributed mass was greater than that of a moving distributed force pattern.

DISCUSSION

Analytical solutions for the beam-type structural member were obtained for the governing differential equation of motion. Clearly, justification was highly achieved when this study was correlated with what obtained in Oni[4], Oni[5], Omolofe and Alimi[10] and Oni and Ogunyebi[11]. Hence, this study has authenticated its results with the presented theoretical solution. Also, it was established that resonance

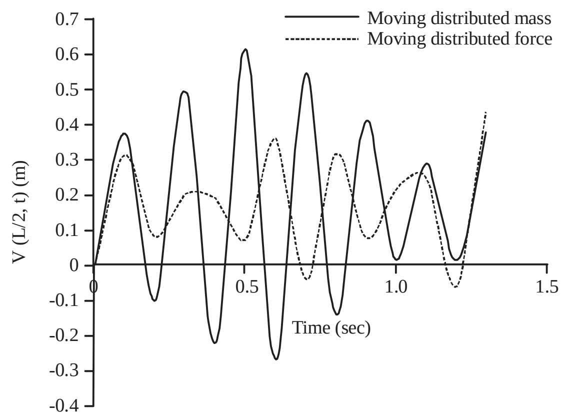

Fig. 10: Comparison of the displacement response of moving of a Simply supported non-uniform Rayleigh beam on variable Pasternak foundation and traversed by moving distributed load for fixed values of N = 10000, So = 30000, Ko = 2000, Ro = 0.2 and of Eo = 0.5

was reached earlier in the moving distributed mass system than in the moving distributed force system and this new findings for the simply supported Rayleigh beam problem can be extended to all variants of boundary conditions for both one-dimensional and two-dimensional problems in the dynamics of structures under moving loads.

CONCLUSION

This work presents the dynamic analysis of finite simply supported non-uniform Rayleigh beam resting on variable bi-parametric elastic subgrade under the action of moving distributed loads. The fourth order partial differential equation governing the system is reduced to second order coupled ordinary differential equations called Galerkin equations are solved by expression Heaviside function in series form, a modification of Struble's asymptotic technique and convolution theory. As the axial force, foundation stiffness, shear modulus and rotatory inertia correction factor increases, the response amplitude of the simply supported uniform Rayleigh beam decreases. Also, for fixed value of axial force, foundation stiffness, shear modulus and rotatory inertia correction factor, the response amplitude for the moving distributed mass problem is greater than that of the moving distributed force as displayed in plotted curves.

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