International Research Journal of APPLIED SCIENCES

Volume 01 | Issue 01 | 2019

SciRange
PUBLICATIONS
www.scirange.com

SciRange Open Access

International Research Journal of Applied Sciences Research Article

pISSN: 2663-5577, eISSN: 2663-5585

Some Volatility Modeling Using New Error Innovation Distribution

[1]Samson Agboola, [1]Hussaini Garba Dikko and [2]Osebekwin Ebenezer Asiribo

[1]Department of Statistics, Ahmadu Bello University, Zaria, Nigeria
[2]Department of Statistics, Federal University of Agriculture, Abeokuta, Nigeria

ARTICLE INFORMATION

Received: October 29, 2018

Accepted: November 25, 2018

Published: January 31, 2019

Corresponding Author:
Samson Agboola,
Department of Statistics,
Ahmadu Bello University, Zaria, Nigeria

ABSTRACT

One of the major assumptions underlying the use of Ordinary Least Square (OLS) in regression analysis is that the variance of the error is constant over time. This is referred to as homoscedasticity. This assumption does not usually hold when dealing with financial series as financial series do exhibit heteroscedasticity. This problem led to the development of heteroscedastic models. The purpose of this study was to propose a new error innovation distribution in estimating some volatility models. A new error innovation distribution is known as Exponentiated Skewed Student-t Distribution (ESSTD). These were compared with other error distributions with an empirical dataset of daily returns from Standard and Poor five hundred (S&P500) index return to validate the best fit and forecasting performance in terms of models and error innovation distributions. The stocks showed evidence of stationarity with the Augmented Dickey-Fuller (ADF) Statistic while ARCH effect statistic using Lagrange Multiplier shows proof of ARCH impact. The estimate of the volatility models was significant at 1 and 5% p-values for the error innovation distributions. The Akaike Information Criteria (AIC) show that the new error distributions outperformed in terms of fitness on GARCH (1,1), GJR-GARCH (1,1), EGARCH (1,1) and TGARCH (1,1) whereas APARCH (1,1) model with SGED outperformed the new error innovation distribution however in prediction performance, the new error innovation distribution shows the small values RMSE.

Key words: Volatility models, error distributions, S&P500, AIC effect, ARCH effect

INTRODUCTION

Error distribution is one of the very important techniques in estimating the parameters of any volatility models which was why Engle[1] planned to use the innovation distribution as a slip-up distribution in estimating his proposed volatility model. However, this error distribution, innovation distribution proposed by Engle[1] has gained additional ground within the estimation of the volatility models, followed by the scholar t-distribution which was proposed by Bollerslev[2]. Moreover, so as to estimate the parameters of those heteroscedastic models, numerous distribution of error innovation were proposed. This was often as a result of error innovation distribution and plays a vital role in estimating the parameters of the heteroscedastic model[3]. There were six kinds of error innovation distributions that have gained quality as mention higher than in volatility modeling[4,5] particularly distribution, skew distribution, student-t distribution, skew student-t distribution, generalized error distribution and skew generalized error[6]. Conjointly worked on the principle of parsimony in modeling statistic mistreatment heteroscedasticity models mistreatment penurious volatility of ARCH, GARCH, EGARCH and TGARCH and Power ARCH (PARCH) models[7]. They also worked on modeling abrupt shift in statistic mistreatment indicator variable on twelve volatility models.

With the limitation found on the prevailing error innovation distribution particularly the flexibility to capture extreme price, serious caudated etc. prompted US to lead off the article by proposing a brand new error distribution from the distribution derived by Dikko and Agboola[7] known as the Exponentiated skew student-t distribution. This was to model some volatility models and estimate their parameters victimization planned error distribution and compare it in terms of fitness and statement analysis victimization in normal and poor five hundred (S&P500) index returns daily dataset.

The ARCH model parameters were estimated using the normal error distribution proposed by Engle[1] for volatility models. The model of GARCH[2] also adopted this normal distribution for the error innovation and used the distribution to estimate volatility models not until the ground break in the estimation that the error term does not followed a normal distribution which make Bollerslev[2] used the student-t distribution in the estimation of volatility models. Generalized Error Distribution (GED) was proposed for error innovation[8] in estimating EGARCH model. Even since there being extension of GARCH models proposed by other researchers.

The objective of the study was to use new error innovation distribution in modeling Standard and Poor 500 index return data and also compare some volatility models and error innovation distributions in terms of fitness and forecasting performance.

MATERIALS AND METHODS

Data for the study: The data for the study covered a period of 2007 to 2017 using daily closing price index.

Return series computation from price:

Let:
$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \quad (1)$$

where, P_t and P_{t-1} are the present and previous closing prices and r_t the continuously compounded return series which is the natural logarithm of the simple gross return.

Stationary test: The return series statistic using Augmented Dickey-Fuller test is illustrated with first differenced series given as:

$$x_t = \phi_1 x_{t-1} \quad (2)$$

$$x_t - x_{t-1} = \varphi_1 x_t - x_{t-1}$$
$$\Rightarrow \varphi_1 - 1 = 0 \text{ or } \varphi_1 = 1$$

Null hypothesis is illustrated as $H_0 : \varphi_1 = 1$

While, alternative hypothesis is: $H_1 : \varphi_1 < 1$ and the statistic is given as:

The Test Statistic (t-ratio):

$$= \frac{\varphi_1^n - 1}{\text{std}(\varphi_1)} = \frac{\sum_{t=1}^{T} P_{t-1} e_t}{\sigma^2 \sqrt{\sum_{t=1}^{T} P_{t-1}^2}} \quad (3)$$

$P_0 = 0$, T is the sample size and φ_1 for each stock.

If the calculated value of t is greater than t critical value then the null hypothesis is rejected.

Test for heteroscedasticity effect: Lagrange multiplier test proposed by Engle[1] will be adopted in testing the presence of heteroscedasticity in the data set. The test F statistic for testing $\alpha_i = 0$ (i = 1,...., m) in the linear regression:

$$\alpha_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \ldots + \alpha_m a_t^2 + \varepsilon_t,\ t = m+1, \ldots, T \quad (4)$$

where, ε_t denote the error term, m is a pre-specified positive integer and T is the sample size.

The null hypothesis is:

$$H_0 : \alpha_1 = \ldots = \alpha_m$$
$$H_a : \alpha_i \neq 0 \text{ for some } i \in \text{some}\{1, \ldots, m\}$$

The test statistic:

$$F = \frac{(SSR_0 - SSR_1)/m}{SSR_1(T - 2m - 1)} \quad (5)$$

where, $SSR_1 = \sum_{t=m+1}^{T} e_t^2, e_t^2$ is the residual of least square of the linear regression and

$$SSR_0 = \sum_{t=m+1}^{T} (a_t^2 - w), w = \frac{1}{T}\sum_{t=m+1}^{T} a_t^2 \quad (6)$$

is the sample mean of a_t^2.

Some volatility models: The GARCH (p, q) model was stated as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \ldots + \alpha_q \varepsilon_{t-q}^2 + \beta_1 \varepsilon_{t-1}^2 + \ldots + \beta_p \varepsilon_{t-p}^2 + r_t \quad (7)$$

where, $\alpha_i > 0$ and $\beta_i > 0$ for all i and j.
The EGARCH (p, q) model was proposed by Nelson[8], formulate the volatility which is as follows:

$$\ln(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{p} \alpha_i \left[\lambda \varepsilon_{t-i} + \gamma \left\{ \left| \varepsilon_{t-i} \right| - \sqrt{\frac{2}{\pi}} \right\} \right] + \sum_{i}^{q} \beta_j \ln(\sigma_{t-j}^2) \quad (8)$$

$\alpha_0, \alpha_i, \gamma, \beta_j$ are the parameters of the model.
Threshold Generalized Autoregressive Conditional Heteroscedasticity (TGARCH) model.
Glosten *et al.*[9] stated the model as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} (\alpha_i + \gamma_i N_{t-i}) \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^{\ 2} \quad (9)$$

$\alpha_0, \alpha_i, \gamma_i, \beta_j \geq 0$

where, N_{t-i} is an indicator for negative ε_{t-i} that is N_{t-i} is 1 if $\varepsilon_{t-i} <$ 0 and 0.

Model selection: Akaike Information Criteria (AIC,) is used in model selection criteria which are given as:

$$AIC = -2\ln(LL) + 2p \quad (10)$$

where, p is the number of parameters in the model and LL is the maximized value of the likelihood function for the model.

- **Forecasting performance**

The Root Mean Squared Error (RMSE) statistic is used for model forecasting. The Statistic is given as:

$$RMSE = \sqrt{\frac{\sum_{t=T+1}^{T+n} \left(\hat{\sigma}_t^2 - \sigma^2{}_t\right)^2}{n}} \quad (11)$$

where, n is the number of steps ahead, T is the sample size, $\hat{\sigma}_t$ and σ_t are the square root of the conditional forecasted volatility and the realized volatility respectively.

- **Maximum likelihood estimator**

The following are the commonly used distributions for error innovation in volatility modeling. Generally, in volatility modeling, the standardized form of the distribution is usually used[10].

Skewed normal distribution:

$$f(x_t) = \frac{1}{\sigma\pi} e^{-\frac{(x_t-\varepsilon)^2}{2\sigma^2}} \int_{-\infty}^{\alpha} \frac{x_t - \varepsilon}{\sigma} e^{-\frac{t^2}{2}} dt, \ -\infty < z_t < \alpha \quad (12)$$

where, ε is the location, σ is the scale and α denotes the shape parameter.

Standardized skewed student t-distribution:

$$f(x_t, \mu, \sigma, v, \lambda) = \begin{cases} bc\left[1 + \dfrac{1}{v-2}\left(\dfrac{b\left(\dfrac{x_t - \mu}{\sigma}\right) + a}{1-\lambda}\right)^2\right]^{-\frac{v+1}{2}}, & x_t < -\dfrac{a}{b} \\ bc\left[1 + \dfrac{1}{v-2}\left(\dfrac{b\left(\dfrac{x_t - \mu}{\sigma}\right) + a}{1+\lambda}\right)^2\right]^{-\frac{v+1}{2}}, & x_t \geq -\dfrac{a}{b} \end{cases} \quad (13)$$

where, v is the shape parameter with $2 < v < \infty$ and λ is the Skewedness parameters with $-1 < \lambda < 1$, μ and σ^2 are the mean and variance of the Skewed student t-distribution.

$$a = 4\lambda c\left(\frac{v-2}{v-1}\right), \ b = 1 + 3\lambda^2 - a^2, \ c = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi(v-2)\Gamma\left(\frac{V}{2}\right)}}$$

Standardized skewed generalized error distribution:

$$f(x_t / v, \varepsilon, \theta, \delta) = \frac{v}{2\theta\Gamma\left(\frac{1}{v}\right)} \exp\left[-\frac{|x_t - \delta|^v}{\left[1 + \text{sign}(x_t - \delta)\varepsilon\right]^v \theta^v}\right] \quad (14)$$

Where:

$$\theta = \Gamma\left(\frac{1}{v}\right)^{0.5} \Gamma\left(\frac{3}{V}\right)^{-0.5} S(\varepsilon)^{-1}, \ \delta = 2\varepsilon S(\varepsilon)^{-1}, \ S(\varepsilon) = \sqrt{1 + 3\varepsilon^2 - 4A^2\varepsilon^2},$$

$$A = \Gamma\left(\frac{2}{v}\right)\Gamma\left(\frac{1}{v}\right)^{-0.5} \Gamma\left(\frac{3}{v}\right)^{-0.5}$$

where, $v > 0$ is the shape parameter, ε is a Skewedness parameter with $-1 < \varepsilon < 1$. $-\infty < x_t < \infty, \ v > 0, \ -1 < \varepsilon < 1, \ -\infty < x_t < \infty.$

Error innovation of Exponentiated Skewed student-t Distribution (ESSTD):

$$\mathrm{InL}(x;u,\lambda)\sigma_t^2 = n\log(\alpha) + nt\log(\lambda) - n\alpha\log 2 + (\alpha-1)\sum_{i=1}^{n}\log\left(1+\frac{\varepsilon_t}{\sigma_t\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2}}\right) - \frac{3}{2}\sum_{i=1}^{n}\log\left(\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2\right) - 0.5\log\left(\sigma_t^2\right) \quad (15)$$

where, $\theta = (\alpha,\lambda,\sigma_t)$ and $\sigma_t^2 = \omega + \sum_{i=1}^{n} a_i\varepsilon_{t-i}^2 + \sum_{i=1}^{n}\beta_i\sigma_{t-i}^2$

RESULTS AND DISCUSSION

Descriptive statistic: Descriptive statistic of the S&P500 index returns data were used. The obtained statistics are shown in Table 1 that the mean return series were negative, positive Skewed and high kurtosis for S&P500 index returns. The return was not normally distributed since the Jarque-Bera statistic p-value is less than 1%. This finding agreed with the previous work[11] of non-normally distributed return series.

Table 1: Summary statistics of standard and poor 500 index return

Statistics	S&P500
Mean	-0.0002
Std. Dev.	0.0129
Skewedness	0.3357
Kurtosis	13.485
Jarque-Bera	12008.37
Probability	0.001
Observations	2610

Table 2: Stationarity test using ADF of S&P500 index returns

Stocks	ADF test statistic	Comment
S&P500	-12.36540	Stationary at level without transformation

1% critical = -3.432219

Table 3: Lagrange multiplier test for ARCH effect

	ARCH effect	F-Statistic	p-value
S&P500 index returns	Lag 1-2	F(2,2390) = 243.93	0.001
	Lag 1-5	F(5,2384) = 150.66	0.001
	Lag 1-10	F(10,2374)= 93.623	0.001

Augmented dickey fuller test for stationarity: The statistic of the ADF shows that the series was stationary since the ADF statistic is greater than 1% critical level. Therefore, there was no need for transformation (Table 2). This finding agreed with the previous studies of Dikko *et al.*[11] and Agboola *et al.*[12] using the ADF test in testing for the stationarity of the return series.

Autoregressive conditional heteroscedasticity (ARCH) effect test: In order to estimate the volatility model an ARCH effect test were carried out to test for the presence in the series using the Lagrange Multiplier F Statistic. The result showed the presence of ARCH effect with p-value level than 1% (Table 3). This finding agreed with the works[1,2,6] using Lagrange Multiplier in testing the presence of ARCH effect which shows significant effect.

Parameters estimation with error innovation distributions on standard and poor 500 (S&P500) index returns: Table 4 and 5 present the parameter estimates of studied volatility models using five (5) error innovation distributions namely; skewed normal, skewed distribution, skewed student-t distribution, skewed generalized error distribution and the proposed Exponentiated skewed student-t distribution using returns from S&P500. The Table 4 shows the estimate using the new proposed error distribution and Table 5 shows the estimate using the existing error distributions. The result shows that the returns exhibit volatility clustering. This was concluded because the GARCH term was significant in most of the models considered ($p<0.05$) and ($p<0.01$). This finding is in line with the work Dikko *et al.*[11] and Agboola *et al.*[12].

Fitness and model selection of some volatility models on standard and poor 500 (S&P500) index returns: Table 6 shows the result of the fitness and model selection using AIC. The GJR-GARCH (1,1), EGARCH (1,1) and TGARCH (1,1) with error innovation distribution of ESSTD outperformed better than the others distributions of error innovation and SGED outperformed others error distribution including proposed error distribution on APARCH (1,1) model. The proposed error innovation ESSTD was found to outperform on four volatility

Table 4: Parameters estimation with new error innovation distribution of standard and poor 500 (S&P500) index returns

Models	Error distribution	ω	α_1	β_1	γ_1	δ	Skewed	Shape(u)
GARCH (1,1)	ESSTD	3.817×10^{-07}**	1.225×10^{-02}	-6.788×10^{-03}**			6.736×10^{-02}**	4.3610**
GJR-GARCH (1,1)	ESSTD	0.06105**	0.00875	0.001961**	0.1000*		0.5530	4.4560**
EGARCH (1,1)	ESSTD	0.50076**	0.10069**	0.08105	0.06155*		0.2698**	1.2651*
TGARCH (1,1)	ESSTD	0.00946*	0.00451**	0.1306	-0.0725		1.7040	1.3170*
APARCH (1,1)	ESSTD	3.792×10^{-07}	0.10000**	0.1021**	0.0995	0.08976	0.03855	7.6382**

p-value significant at: 5%*, 1%** and 10%***

Table 5: Parameter estimation of some volatility model with three (3) error innovation distributions on S&P500 index returns

Models	Error	ω	α_1	β_1	γ_1	δ	Skewed	Shape
GARCH (1,1)	SSTD	1.781×10^{-06}***	1.301×10^{-01}***	8.671×10^{-01}			1.099***	5.018***
	SNORM	2.254×10^{-06}***	1.169×10^{-01}***	8.656×10^{-01}			1.172 **	
	SGED	2.091×10^{06}***	$1.259\times10\text{-}^{01}$***	8.624×10^{-01}***			1.052***	1.173***
GJR-GARCH (1,1)	SSTD	0.00002***	0.2492***	0.8845***	-0.2861**		1.1651**	5.5185**
	SNORM	0.00002***	0.2007***	0.8944***	-0.2300***		1.2036***	
	SGED	0.000002***	0.2360***	0.8827***	-0.2689***		1.1244***	1.2603***
EGARCH (1,1)	SSTD	-0.21282***	0.21282**	0.97706**	-0.12412**		1.1764**	5.5422**
	SNORM	-0.23031***	0.17975**	0.97431**	0.11866**		1.22533**	
	SGED	-0.24797**	0.20479***	0.97327***	0.12999**		1.14487**	1.27319***
TGARCH (1,1)	SSTD	0.000002	0.13037	0.86715**			1.0991**	5.0139
	SNORM	0.000002	0.11690*	0.86596***			1.17207**	
	SGED	0.000002	0.12573	0.86271***			1.0524-***	1.17286***
APARCH (1,1)	SSTD	0.00002	0.05980	0.86703	-0.9996*	2.000***	1.1489**	5.7842**
	SNORM	0.000002	0.05116	0.8780***	-0.9959**	2.000**	1.2005**	
	SGED	0.000002	0.06014	0.8651**	0.9984*	2.000**	1.1141**	1.2524**

p-value significant at: 5%*, 1%** and 10%***

Table 6: Fitness and model selection using AIC on S&P500 index returns

Models	Error	LL	AIC
GARCH (1,1)	SSTD	8456.201	-6.4752
	SNORM	8393.955	-6.4283
	SGED	8476.708	-6.4909
	ESSTD	148400.191	-13.3623
GJR-GARCH (1,1)	SSTD	8521.264	-6.5243
	SNORM	8464.151	-6.4813
	SGED	8531.827	-6.5324
	ESSTD	447704.856	-14.0237
EGARCH (1,1)	SSTD	8530.299	-6.5313
	SNORM	8472.271	-6.4876
	SGED	8538.083	-6.5372
	ESSTD	47009.9981	-9.5162
TGARCH (1,1)	SSTD	8456.228	-6.4753
	SNORM	8393.938	-6.4283
	SGED	8476.710	-6.4910
	ESSTD	42954.5434	-9.3357
APARCH (1,1)	SSTD	8515.994	-6.5203
	SNORM	8459.808	-6.4780
	SGED	8528.098	-6.5296
	ESSTD	14029.671	-5.0978

Bolded values are the highest value of likelihood function and the least value of AIC

Table 7: Forecasting evaluation of some volatility models using standard and poor 500 (S&P500) index returns

Models	Error distributions	RMSE
GARCH (1,1)	SSTD	1.082302
	SNORM	0.5294797
	SGED	0.4134991
	ESSTD	0.0000510
GJR-GARCH (1,1)	SSTD	0.02316053
	SNORM	0.2594925
	SGED	0.1145864
	ESSTD	0.0000536
EGARCH (1,1)	SSTD	0.01463527
	SNORM	0.3109253
	SGED	0.09638982
	ESSTD	0.0000057
TGARCH (1,1)	SSTD	1.082566
	SNORM	0.5224235
	SGED	0.4016423
	ESSTD	0.250047
APARCH (1,1)	SSTD	0.009737621
	SNORM	0.1412954
	SGED	0.05158674
	ESSTD	0.000611

Bolded values are the highest value of likelihood function

models as revealed by its least value of Akaike Information Criteria (AIC) for the index returns.

Forecasting performance of estimated volatility models on S&P500 index returns: The result showed that GARCH (1,1), GJR-GARCH (1,1), EGARCH (1,1), TGARCH (1,1) and APARCH (1,1) models performance in term of forecasting under the new error innovation distribution (ESSTD) (Table 7). This finding was in agreement with the previous study[12] adopting the principle of parsimony showed that GARCH (1,1), GJR-GARCH (1,1), EGARCH (1,1), TGARCH (1,1) outperformed except in this study they did not conduct their study on APACH model.

CONCLUSION

It was concluded that in this article, compared various error innovation distributions with new error innovation distribution in estimating the parameters of five (5) GARCH models and its extension. The empirical result showed a negative return in the mean, high kurtosis and positive skewness and it was stationary without transformation. The data also showed evidence of ARCH effect and the parameters estimation showed most coefficients in the models were significant at 1, 5 and 10%. From the results obtained showed on model selection using the AIC, the ESSTD fitted better in the GARCH, GJR-GARCH, EGARCH and TGARCH model while SGED in fitness, the APARCH model outperformed compared to other error distributions while the forecasting evaluation only showed that ESSTD outperformed the existing distributions with least RSME.

ACKNOWLEDGMENTS

This study acknowledge the support of standard and poor 500 stock market for providing daily return data to test and validate the new proposed error distributions and also appreciate the Department of Statistics, Ahmadu Bello University for providing the ground for this paper and the entire Department of Economic, Ahmadu Bello University, Zaria- Nigeria.

REFERENCES

1. Engle, R.F., 1982. Autoregressive conditional heteroscedasticity with estimates of the varaince of U.K. inflation, Econometrica, 50: 987-1008.
2. Bollerslev, T., 1986. Generalized autoregressive conditional heteroscedasticity. J. Econometric, 31: 307-327.
3. Su, C., 2010. Application of EGARCH model to estimate financial volatility of daily returns: The empirical case of China. University of Gothenburg, Master Degree Project No. 2010, pp: 1-32.
4. Bali, T.G., 2007. Modeling the dynamics of interest rate volatility with skewed fat-tailed distribution. Ann. Oper. Res., 151: 151-178.
5. Shamiri, A. and Z. Isa, 2009. Modeling and forecasting volatility of the malaysian stock market. J. Math. Stat., 5: 234-240.
6. Agboola, S., U.M. Sani and I. Danjuma, 2015. Adopting principle of parsimony in modeling time series using heteroscedasticity models. R. J. Econ., 3: 1-10.
7. Dikko, H.G. and S. Agboola, 2017. Statistical properties of exponentiated skewed student-t distributions. Niger. Assoc. Math. Phy., 4: 251-260.
8. Nelson, D., 1991. Conditional heteroskedasticity in asset returns: A new approach. Econometrica, 59: 349-370.
9. Glosten, L., R. Jagannathan and D. Runkle, 1993. On the relation between expected value and the volatility of normal excess returns on stocks. J. Finance, 48: 1779-1801.
10. Diethelm, W., C. Yohan and L. Luksan, 2014. Parameter estimation of ARMA models with GARCH/APARCH error An R and S plus software implementation. J. Stat. Software, 10: 1-41.
11. Dikko, H.G. O.E. Asiribo and S. Agboola, 2015. Modeling abrupt shift in time series using indicator variable: Evidence of Nigerian insurance stock. Int. J. Finance Account., 4: 119-130.
12. Agboola, S., U.M. Sani and I. Danjuma, 2015. Adopting principle of parsimony in modeling time series using heteroscedasticity models. Res. J. Econ., 3: 1-10.